

Iterative Detection with Soft Decision in Spectrally Efficient FDM Systems

Seyed Javad Heydari, Mahmoud Ferdosizade Naeiny, Farokh Marvasti

Abstract—In Spectrally Efficient Frequency Division Multiplexing (SEFDM) systems the input data stream is divided into several adjacent subchannels where the distance of the subchannels is less than that of Orthogonal Frequency Division Multiplexing (OFDM) systems. Since the subcarriers are not orthogonal in SEFDM systems, they lead to interference at the receiver side. In this paper, an iterative method is proposed for interference compensation for SEFDM systems. In this method a soft mapping technique is used after each iteration block to improve its performance. The performance of the proposed method is comparable to that of Sphere Detection (SD) which is a nearly optimal detection method.

Index Terms—OFDM, SEFDM, Iterative Method, Soft Mapping

I. INTRODUCTION

In recent years, Orthogonal Frequency Division Multiplexing (OFDM) has attracted significant research interest as a potential solution for high-rate data service demands over wireless channels [1], [2]. In OFDM systems, the total bandwidth is divided into several orthogonal narrowband subcarriers. Since the subcarriers have a 50 percent overlapping, OFDM system leads to a high spectrum efficiency comparing to the ordinary Frequency Division Multiple Access (FDMA) systems. The lower data rate in each subcarrier makes the OFDM systems have a great advantage to combat Inter-Symbol Interference (ISI) and enable high degree of flexibility of resource allocation among users. However, demands for broadband wireless applications have grown significantly in recent years and a reliable transmission and reception of high data rate information over a lower bandwidth has drawn a lot of attention. Rodrigues and Darwazeh in [3] and Xiong in [4] introduced a multicarrier system that occupies half of the bandwidth of an equivalent OFDM, but the detection was possible for real alphabets (e.g. BPSK) only. Later, Rodrigues and Darwazeh in [5] and Hamamura and Tachikawa afterwards in [6] proposed spectrally efficient FDM systems in which subcarrier frequency separation is smaller than the inverse of the FDM signaling period, so they deliberately violated the orthogonality principle between carriers. Consequently, these systems have considerable bandwidth efficiency in comparison to that of OFDM systems.

Rusek and Anderson have shown in [7] that the concept of reducing the frequency separation between orthogonal subcarriers is a dual case of the Mazo's time-domain transmission

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technique. Mazo aims to transmit information faster than the Nyquist rate [8], [9]. In addition, they have proved that in presence of AWGN, the frequency separation of the subcarriers can be reduced up to a limit, which is the dual of the so-called Mazo's limit. As a consequence, if this limitation is taken into account, no degradation is expected in the performance of spectrally efficient FDM systems in comparison to OFDM systems. Notwithstanding, the reliable detection of the information of such FDM systems is still a very challenging issue since the optimal maximum likelihood detection is very complex. In [7], it has been shown that it is safe to increase the signaling rate by 20% without expecting any performance degradation.

Transmission of the SEFDM signal can be performed with the standard Inverse Fast Fourier Transform (IFFT) [10], [11]. However, detection of SEFDM signals is challenged by the need to extract the original signal from the inter-carrier interference. The optimal detection of the SEFDM signal requires brute force ML which can become extremely complex [5]. On the other hand, using linear detection techniques such as Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) constrains the size of the SEFDM system and the level of bandwidth savings due to the ill-conditioning of the system caused by the orthogonality collapse [12]. The Truncated Singular Value Decomposition (TSVD) was proposed as an efficient tool to overcome this deficit [13], but its performance was far from Optimum detector. Therefore, Sphere Decoder (SD) was proposed in [14] to provide ML performance at a much-reduced complexity. Nevertheless, the SD complexity varies greatly depending on the noise. Convex Optimization tools are also used to achieve a reliable detector [15], but this method suffers from the same problems as the linear ones. In [16], quasi-optimal detector combining Semi-definite Programming and SD was proposed; however, the complexity still remains variable. In [17], authors proposed the use of the Fixed-complexity Sphere Decoder (FSD) algorithm for the detection of SEFDM signal. The FSD algorithm could no longer guarantee an optimum solution; However, its sub-optimality may be traded-off with complexity. A Precoding strategy that greatly simplifies the detection of the signal was proposed in [18]. The strategy facilitates simpler detection for the same bandwidth savings as an equivalent uncoded SEFDM system. However, it also suffers from ill-conditioning of the system.

In this paper, an iterative method is proposed whose performance is comparable with that of SD method with lower complexity. This method is a modified version of the general iterative method introduced in [19]. In the modified version,

a soft mapping operation is used in order to decide some symbols after each iteration. It is proposed that the soft mapping parameter is adaptively changed after each iteration block. In section II the system model of SEFDM systems is discussed and in section III the proposed iterative method is introduced. Then, in section IV the performance of the iterative method is evaluated by simulations.

II. SYSTEM MODEL

In the SEFDM system, the input stream of the complex symbols is divided into N parallel low data rate streams. The generated symbol streams are modulated by different carrier frequencies. If $S(n)$ is the complex symbol of the n th data stream, then the time domain baseband signal can be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} S(n) e^{j2\pi n \Delta f t}, \quad 0 \leq t < T, \quad (1)$$

where Δf is the spacing between the adjacent carrier frequencies. The main difference between SEFDM and OFDM systems is that the distance of carrier frequencies in SEFDM systems is only a fraction of the inverse of the FDM symbol period T , i.e.

$$\Delta f = f_k - f_{k-1} = \frac{\alpha}{T}, \quad \alpha < 1 \quad (2)$$

As a result, the required bandwidth is reduced by a factor of $1 - \alpha$, at the expense of the loss of orthogonality between carriers. The time domain samples of SEFDM signal are as follows:

$$X(k) = x\left(\frac{kT}{N}\right) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} S(n) e^{j\frac{2\pi\alpha n k}{N}}, \quad 0 \leq k < N - 1, \quad (3)$$

If $\mathbf{X} = [X(0), X(1), \dots, X(N - 1)]^T$ and $\mathbf{S} = [S(0), S(1), \dots, S(N - 1)]^T$ then the above equation can be written in the matrix form of $\mathbf{X} = \mathbf{FS}$, where \mathbf{F} is $N \times N$ matrix with the elements of

$$f_{k,n} = \frac{1}{\sqrt{N}} e^{j\frac{2\pi\alpha n k}{N}} \quad (4)$$

according to [11], SEFDM signal can be produced using IFFT operation. It is assumed that at the receiver side the transmitted signal, $x(t)$, is received with an Additive white Gaussian Noise (AWGN); $r(t) = x(t) + n(t)$. To extract the sufficient statistics from $r(t)$, N correlators are used at the receiver side. The output of the i th correlator is calculated by

$$R(i) = \int_0^T r(t) b_i^*(t) dt, \quad i = 0, 1, \dots, N - 1 \quad (5)$$

In [16] orthonormal function $b_i(t)$, $i = 0, 1, \dots, N - 1$ were used to prevent noise coloring and loss of information, but in this paper, we do not limit ourselves to use only orthonormal bases and like [20], the kernels used for modulation at the transmitter are used as bases of the receiver projection. As a result, we can benefit from a simple FFT block instead of a correlator bank. If the output of the correlator bank is shown by the vector $\mathbf{R} = [R(0), R(1), \dots, R(N - 1)]^T$, then

$$\mathbf{R} = \mathbf{MS} + \mathbf{N} \quad (6)$$

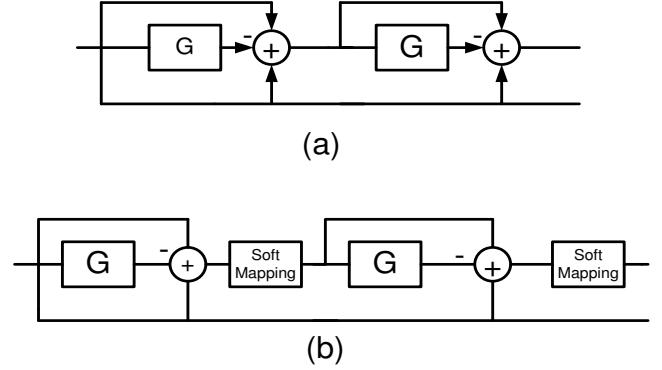


Fig. 1. Block diagram of the iteration method for distortion compensation
a) Original method b) modified iterative method

where the elements of the matrix \mathbf{M} and the vector \mathbf{N} are defined by

$$\begin{aligned} M_{ik} &= \int_0^T b_i^*(t) e^{j2\pi k \Delta f t} dt \\ N_i &= \int_0^T n(t) b_i^*(t) dt \end{aligned} \quad (7)$$

The main problem is to detect the vector of the transmitted symbols \mathbf{S} from the vector \mathbf{R} .

III. ITERATIVE DETECTION METHOD

A modified iterative method is proposed to be used for detection of the transmitted symbols in SEFDM systems. The iterative technique was first proposed by Marvasti [19] for the cancellation of the interpolation distortion and then it was shown that this method could be used for the compensation of the distortion caused by linear or nonlinear operations [21], [22]. If a signal \mathbf{S} is distorted by the distortion operation \mathbf{G} to produce $\hat{\mathbf{S}} = \mathbf{GS}$, then the signal \mathbf{S} can be reconstructed from the distorted version, $\hat{\mathbf{S}}$, as follows (Fig.1.a)

$$\mathbf{S}_n = \lambda \mathbf{S}_0 + (1 - \lambda \mathbf{G}) \mathbf{S}_{n-1}. \quad (8)$$

where λ is the relaxation parameter, \mathbf{S}_n is the output after n iterations and $\mathbf{S}_0 = \mathbf{GS}$. It has been shown in [19] that if the power of distortion, $\|\mathbf{S} - \mathbf{GS}\|_2$, is less than the power of the signal, $\|\mathbf{S}\|_2$, then the iterative method converge to the desired signal after an infinite number of iterations. It has also been shown that if \mathbf{G} is a nonlinear distortion operation, the proper selection of the relaxation factor of λ can speed up the convergence.

In this paper, a modified version of this method is used to compensate for the interference of the carriers in SEFDM systems. In other words, FDM modulation and correlation bank in the SEFDM systems are considered as a distortion function, \mathbf{G} , and the iterative method is applied to compensate for the distortion, i.e. $\mathbf{GS} = \mathbf{MS}$. It can be shown that the value of Signal to Distortion Ratio (SDR) depends on the number of carriers, N , and the frequency separation α . The closer to 1 the value of α or the more the number of carriers,

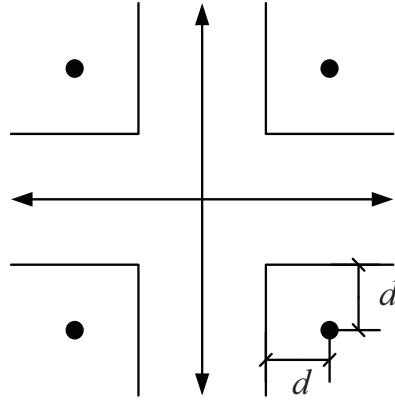


Fig. 2. Soft mapping areas in 4-QAM modulation with parameter d

the larger the SDR. Therefore, the iterative method converges faster and to a better output.

It is extra information that we know the elements of the desired vector \mathbf{S} belong to a constellation. Thus, in the modified version of the iterative method at the output of each iteration block, the points are mapped to the constellation points (as shown in Fig.1.b). This operation has the effect of eliminating distortion and noise after each iteration, but it may also have a disadvantage. If hard mapping is used all the symbols are mapped to the nearest constellation point and it may neutralize the effect of the iterations because it prevents the movement of the symbols to the proper answer. Thus, a soft mapping method is proposed instead of it. In this mapping strategy, the symbols in the neighboring area of the constellation points are mapped and the other ones are left unchanged. The procedure of the mapping can be written by:

$$Q(\mathbf{S}) = \begin{cases} C_l & \text{if } \mathbf{S} \in A_l, \quad l \in \{1, 2, \dots, L\} \\ \mathbf{S} & \text{if } \mathbf{S} \notin A_l, \end{cases} \quad (9)$$

where C_1, C_2, \dots, C_L are the constellation points and A_l is the neighboring area around the constellation point C_l . For 4-QAM modulation the neighboring areas are defined by the parameter d as shown in Fig.2. The main advantage of this mapping is that the points which are uncertainty and outside the defined areas in Fig.2, are kept unchanged to be modified by the iteration and the points that are very close to a constellation point are decided. After each iteration, the number of undecided symbols is reduced. Thus, the modified iteration method can be described by:

$$\begin{aligned} \mathbf{S}_n &= Q\left(\lambda \mathbf{S}_0 + (1 - \lambda \mathbf{G}) \mathbf{S}_{n-1}\right) \\ \mathbf{S}_0 &= \mathbf{F}^{-1} \mathbf{R} \end{aligned} \quad (10)$$

Another method which is proposed in this paper is to adjust the mapping areas in Fig.2 dynamically. In this scheme the neighboring areas $A_l, l = 1, 2, \dots, L$ are expanded after each iteration. As it can be shown, a simple iteration without mapping has a very moderate performance. Moreover, soft mapping has a better performance than that of hard mapping. As a result, it should be noted that soft mapping has a very

influential role in our method in order to achieve the desired BER.

IV. ANALYSIS OF COMPLEXITY

The main advantage of the proposed method, in comparison to SD and ML methods introduced in [5], is its low complexity. The complexity of these methods can be evaluated by number of Real Additions (RA) and Real Multiplications (RM). In this section, the order of complexity according to this criterion is computed for ML, SD and the proposed iterative methods.

In ML method, the following metric must be calculated for L^N different possible vectors \mathbf{S} :

$$\|\mathbf{R} - \mathbf{MS}\|_2 \quad (11)$$

The computation of \mathbf{MS} for each vector \mathbf{S} consists of an N/α points IFFT and an N/α points FFT which needs $2N/\alpha \log_2(N/\alpha)$ RMs and $6N/\alpha \log_2(N/\alpha)$ RAs [23]. Moreover, $4N$ RAs and $2N$ RMS are needed to obtain L2 norm of the error vector in (11). Consequently, $2L^N N/\alpha(3 \log_2(N/\alpha) + 2\alpha)$ RAs and $2L^N N/\alpha(\log_2(N/\alpha) + \alpha)$ RMs should be performed for every data block.

In SD method, the metric (11) is calculated for some possible vectors \mathbf{S} which are in a sphere of radius d defined by $\|\mathbf{R} - \mathbf{MS}\|_2 \leq d^2$. In this method, the complexity depends on the number of nodes in this sphere. This number is a random variable which depends on SNR and the radius d . Thus, the expected complexity of SD method can be calculated. In [24], the expected complexity of SD method has been derived in a general form. It has been seen that, the expected number of operations is in order of $L^{\gamma N}$, where $0 < \gamma < 1$ is a small factor that is dependent on SNR and d . However, for large values of SNR, the factor $\gamma \ll 1$, and therefore, $L^{\gamma N}$ is close to 1 when N is small. This means that for large values of SNR and small N , the complexity is dominated by polynomial terms, which is consistent with the results of [25], but in low SNR regimes, it is NP hard. As a result, the numbers of the required RAs and RMs in SD method are like those of the ML method except that L^N must be replaced by $L^{\gamma N}$.

In the proposed method, at each iteration the operation \mathbf{G} must be performed which consists of an N -points IFFT and an N -points FFT. After calculation of the term \mathbf{GS}_{n-1} , the other operations in (10) need $4N$ RAs and $2N$ RMs. For soft mapping, which is performed at the end of every iteration block, the symbols must be compared with all of the constellation points. If we assume that the complexity of this comparison is equivalent to the complexity of one RA, then the complexity of soft mapping is equivalent to that of LN RAs. As a result, in every iteration, $N(L+4+6/\alpha \log_2(N/\alpha))$ RAs and $N(2/\alpha \log_2(N/\alpha) + 2)$ RMs are performed.

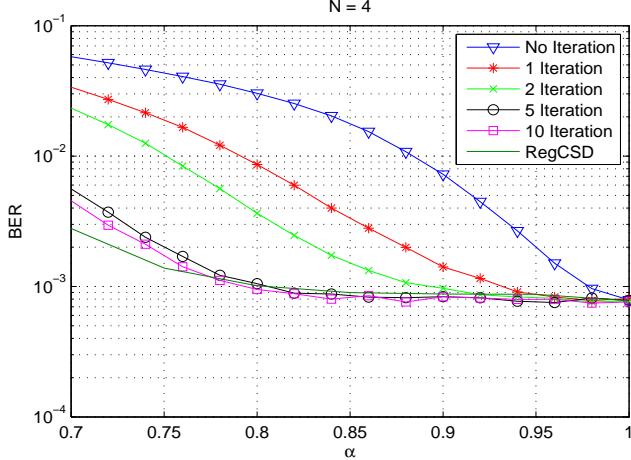


Fig. 3. BER versus α for SEFDM system with $N = 4$ carriers and different detection methods: SD and the proposed method with 1, 2, 5 and 10 iterations.

V. SIMULATION RESULTS

To evaluate the performance of the proposed iterative method, SEFDM systems with 4-QAM modulation and different numbers of carriers, N , and different values of the parameter α have been simulated. Regularized Complex SD (RegCSD) is used as a near optimal method for comparison. Figures 3, 4 and 5 show the Bit Error Rate (BER) versus α for $N = 4$, $N = 8$ and $N = 16$, respectively. The SNR is fixed at 10dB in these figures. In this simulation, the parameter d for soft mapping starts from 1 at the first iteration and is reduced linearly to 0. As it can be seen, for $N = 4$ the performance of the proposed iterative method with 5 iterations is very close to that of near optimal SD method, while (as shown in table I) the iterative method has a lower complexity than that of the SD method. It is clear that the BER is increased when α is decreased. This is more evident when the number of subcarriers is increased from $N = 4$ to $N = 16$. The most important result of these figures is that for $\alpha > 0.85$ performance of iterative method is identical to SD one.

Figure 6 shows BER versus SNR for different values of α and $N = 8$ for proposed method after 10 iterations. In this figure the BER performance of OFDM system ($\alpha = 1$) has also been shown. As it is clear from this figure, the performance degradations in comparison to OFDM systems are 1dB, 2.2dB and 5dB for $\alpha = 0.9$, 0.85 and 0.8, respectively. Thus, the performance of the proposed method is acceptable for the overlapping factors of less than 15%. Figure 7 shows the BER performance of the proposed method for $N = 8$ and $\alpha = 0.85$ versus SNR. It is obvious that the proposed method effectively reduce the BER at the receiver side and can compete SD method, specially in low SNR regime.

Table I compares the ratio of simulation time for the SD method over a single iteration of our proposed method. Since our method converges after 10 iteration, it is apparent that we gain a great advantage in term of complexity by our method.

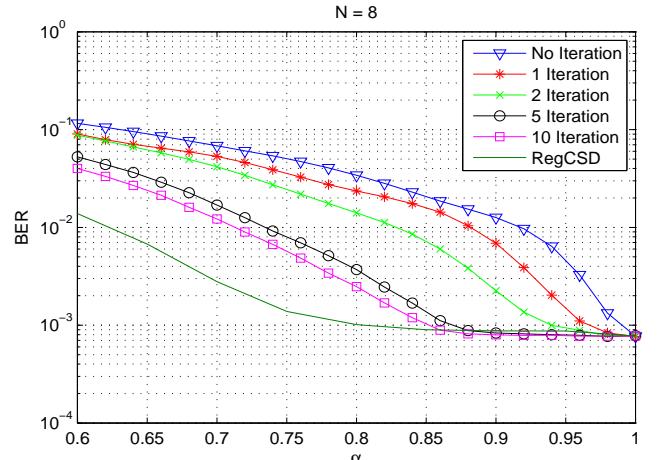


Fig. 4. BER versus α for SEFDM system with $N = 8$ carriers and different detection methods: SD and the proposed method with 1, 2, 5 and 10 iterations.

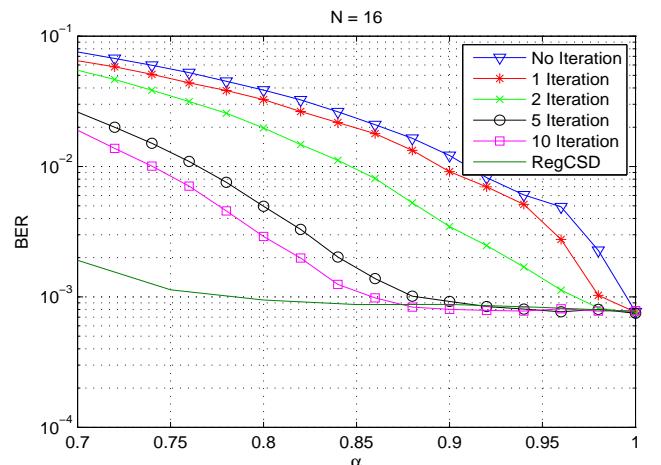


Fig. 5. BER versus α for SEFDM system with $N = 16$ carriers and different detection methods: SD and the proposed method with 1, 2, 5 and 10 iterations.

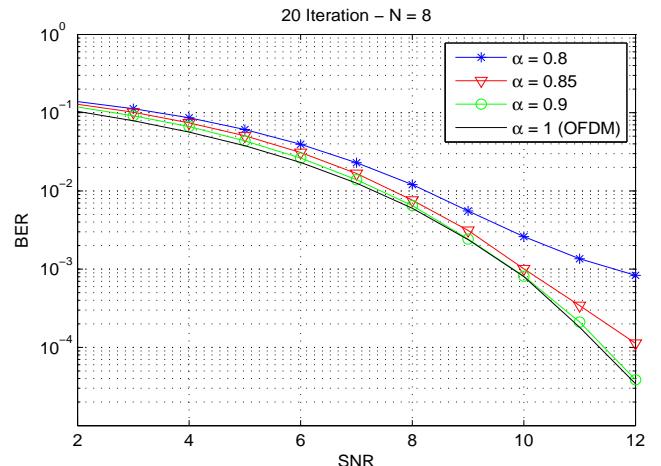


Fig. 6. BER versus SNR for SEFDM system with $N = 8$ carriers and different detection methods: SD and the proposed method with 20 iterations.

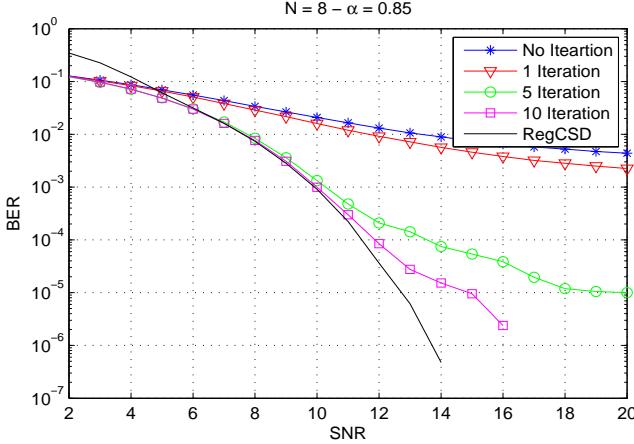


Fig. 7. BER versus SNR for SEFDM system with $N = 8$ carriers and different detection methods: SD and the proposed method.

TABLE I
SIMULATION TIME RATIO γ OF SD OVER A SINGLE ITERATION

γ	$\alpha = 0.8$	$\alpha = 0.85$	$\alpha = 0.9$
$N = 4$	170	160	155
$N = 8$	710	645	640
$N = 16$	4800	4250	4160

VI. CONCLUSION

In this paper, a modified version of an iterative method was proposed for interference reduction in SE-FDM systems. In this method, after each iteration the output is mapped to the constellation points using a soft mapping operation. The soft mapping operation is dynamically changed to the hard mapping after each iteration by changing the decision areas adaptively. The performance of the proposed method was compared with the near optimal SD method. Simulations show that this method has approximately the same performance as that of SD method with much lower complexity when the overlapping is less than 15%.

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